# Self-Excitation of a Chain of Gunn Diodes Connected by Transmission Lines

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**Abstract:** Self-excitation of a chain of Gunn diodes connected by transmission lines and radiating the waves into an infinite open line has been simulated. Simulations are compared with analytical treatment developed for a single-diode circuit with a stub. In a small-signal mode, a multi-frequency spectrum of oscillations is well predicted by a generalized concept of zero total impedance while in a nonlinear mode the spectrum is substantially different, being more complicated though, typically, without continuous component. Long-lasting transients or possibly even genuine dynamical chaos develop, however, in a chain of a few diodes with a small or no load resistance.

# 1. Introduction

Electromagnetic self-excitation in an open system that consists of a one-dimensional chain of active circuits of Gunn diodes connected by sections of transmission line (TL) of length  $d_n$ , with waves radiated from the system into an infinite section of TL (Fig. 1), has been simulated. Conventional analysis tools available for microwave circuits [1], [2] are usually not rigorous enough for such problems while generic numerical methods [3] require significant computational resources. A one-dimensional system considered in this paper provides a link between the two approaches. A chain of a few active devices in a long TL presents a simple model of an open active system with distributed elements that can be used for both the detailed analytical study and rigorous numerical investigations.

# 2. Basic Equations

We consider the chain of Gunn diodes similar to those in [4] but with active circuits as shown in Fig. 1 (c). Unlike the parallel connection of the diodes studied in [4] (Fig.1, b) that partially shunted the high-frequency components, sequential connection here (Fig. 1, c) stimulates the propagation of high-frequency waves. In both cases, the effects depend on the TL parameters, particularly, on the lengths of the TL sections between the diodes. Each diode is simulated as an active element with negative differential resistance (NDR) in some range of the diode voltage  $E_G$  and with the parasitic capacitance C (we use the notations introduced in [4]). This model corresponds to the limited space-charge accumulation (LSA) mode of generation of Gunn diodes that can be realized in a reasonably broad frequency band (up to a decade), being imposed by sufficiently high intrinsic frequencies of the resonant circuits. The equations for the voltages and currents in the circuits and in the TL are obtained in a way similar to [4]. With account of the actual type of circuits, the equations for the chain of N diodes are reduced to the set of 2N ordinary differential equations of the first order with N time delays  $\delta_n = d_n$ .

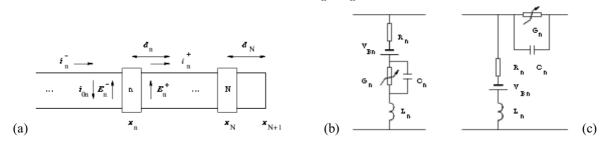


Figure 1. Gunn diode circuits in a transmission line.

In the case of a single diode N = 1 with a stub of length d, we can apply the concept of zero impedance condition Z = 0 [1], [2] to find the frequencies of self-oscillations and to verify this concept against exact numerical solutions. In this way, we obtain the following equation for the angular frequency  $\omega$ 

$$\begin{split} Z_{d}^{2}(\omega) \; \omega \omega_{L}(\omega^{2}Z_{G}^{2} + \omega_{C}^{2}) + Z_{d}(\omega) \{ \omega^{4}Z_{G}^{2} + \omega^{2}[(\omega_{L}Z_{G} - \omega_{C})^{2} + \omega_{L}^{2}Z_{G}^{2}(R^{2} - 1)] + \omega_{0}^{4}R^{2} \} + \\ \omega \omega_{C}Z_{G}^{2}(\omega_{0}^{2} - \omega_{L}^{2}R^{2} - \omega^{2}) = 0 \end{split} \tag{1}$$

where  $Z_S = i Z_d(\omega) = i \tan(\omega d)$  is the stub impedance,  $\omega_L$  and  $\omega_C$  are the characteristic frequencies associated with the circuit inductance L and capacitance C, R is the resistance normalized by the TL intrinsic impedance  $Z_0$  (as all the impedances are),  $\omega_0^2 = \omega_L \omega_C$ , and  $Z_G$  is the diode impedance at the operating point  $E_{G0}$ .

Roots of Eq. (1) with proper treatment of singularities and spurious solutions to satisfy the condition Z=0 define the spectrum of oscillations and, eventually, the effective impedance of the radiating TL section (the antenna radiation resistance)  $Z_r$ . If there is no stub (d = 0), Eq. (1) has a single frequency solution independent of  $Z_G$ ,

$$\omega = \omega_0 \sqrt{1 - R^2 \omega_L / \omega_C} , \qquad (2)$$

with  $Z_r = -Z_G \omega_C / (\omega_C + \omega_L Z_G R)$  (here  $Z_G < 0$  and  $Z_r > 0$  if  $E_{G0}$  is in the NDR domain). If, however, a stub is connected (d  $\neq$  0), Eq. (1) defines a multi-frequency spectrum that essentially depends on the diode impedance.

#### 4. Numerical Results

For the numerical simulations, we choose the same parameters of the devices as specified in [4, 5]. In the case of N = 1, we study the circuits with the stubs of zero length (d = 0, compact open circuits) and of nonzero lengths (extended circuits of size d). In a circuit with no stub (d = 0), when  $G_0$  is just sufficient for the self-excitation, we obtain small oscillations (u ~ 0.1) at a single frequency predicted by Eq. (2) as shown in Fig. 2 (a) (R= 0, 5, 9 at  $\omega_L$ = 0.1,  $\omega_C$ =10 for the solid, dashed and dotted lines, producing  $\omega$  = 1, 0.866, and 0.436, respectively). With increasing the diode admittance in the domain of oscillatory solutions, the basic frequency decreases approaching zero and higher harmonics appear. The spectrum remains, however, rather simple, with just a few harmonics of relatively small amplitude being present, Fig. 2 (b).

In a circuit with a stub, we observe small oscillations at multiple frequencies as expected from Eq. (1). Fig. 2 (c) shows the spectrum of small-signal radiation from the circuit with the stub of length  $d_0 = \lambda_0/2$  resonant with the frequency  $\omega_0 = 1$  ( $Z_d(\omega_0) = 0$ ,  $d_0 = \pi$ ) when  $\omega_L = 0.1$ ,  $\omega_C = 10$ ,  $G_0 = 2$ , and  $G_0 = 0$ . The spectrum is consistent with the roots and singularities of the left-hand part of Eq. (1) as Fig. 2 (d) shows. The excitation of circuits with the stub appears at very small  $G_0$ , e.g., even with  $G_0 < 0.2$  in this example compared to  $G_0 \ge 11.2$  ( $G_0 = 0.2$ ) in a similar circuit with no stub in Fig. 2 (a) (if  $G_0 = 0.2$ ) Ohm, GaN diode parameters in [5] correspond to  $G_0 = 13$ ).

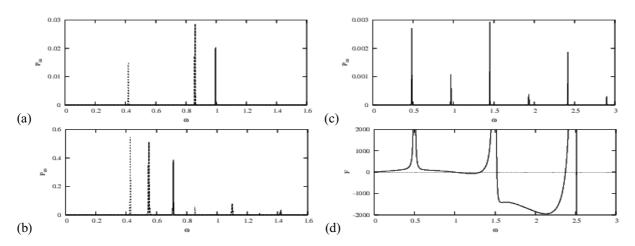
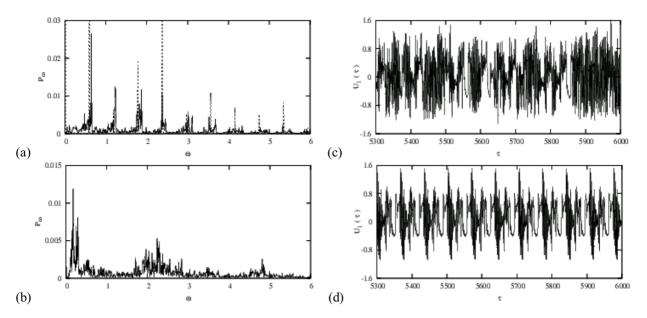


Figure 2. Spectrum of radiation from a single circuit when (a) d = 0 at small  $G_0 \sim 11...12$ , (b) d = 0 at large  $G_0 \sim 14...16$  (R= 0...5), (c)  $d = \lambda_0 / 2$  at small  $G_0$  ( $G_0 = 2$ ) and (d) the left-hand part of Eq. (1) in the case (c).

In a nonlinear mode, the spectrum of radiation from a circuit with the stub has little in common with the spectrum predicted by Eq. (1) at the relevant value of  $G_0$  (both spectra vary significantly with  $G_0$ ). With increasing  $G_0$ , the oscillations remain multi-frequency, though the basic frequency decreases like in the case of a circuit with no stub.

More interesting effects are observed in the chain of N circuits. With increasing N, we observe broadening of spectral lines alone with increasing the number of lines as shown in Fig. 3 (a). The effect appears even in a regular chain of identical circuits with TL sections of the same length d (a stub of the last circuit is d/2) if R=0 (solid curve) while the lines remain narrow if there is a significant resistance in each circuit (e.g., R=5, dotted lines).



**Figure 3**. Spectrum of radiation from a chain of N = 4 circuits similar to those in Fig. 2 (c) when (a)  $d_n = d = 5$  and (b)  $d_1 = 6.91$ ,  $d_2 = 3.77$ ,  $d_3 = 8.17$ ,  $d_4 = 3.14$  (in both cases,  $G_0 = 13$ ). Radiated waveform in the latter case if (c) R = 0.1 (spectrum b) and (d) R = 0.5 (spectrum is very similar to b).

If the chain is irregular, e.g., with different lengths of TL sections, the broadening of spectral lines is significant, the spectrum could be quasi-continuous, Fig. 3 (b), and the radiated wave rather chaotic, Fig. 3 (c). This effect could signify the transition to the dynamical chaos, though it requires careful examination. In such cases, the waveform looks rather chaotic during the whole long time of computation if R is small (up to  $\tau = 8000$  computed at R = 0.1 in Fig. 3, c) but switches to a regular pattern after an extended period of time  $\tau_s$  if R is not so small ( $\tau_s = 1400$  if R = 0.5, Fig. 3 d). Both spectra, however, are remarkably similar, having a distinctive quasi-continuous component.

## 6. Conclusions

A transmission line with active circuits serves as a model of an oscillator with distributed parameters emitting the radiation in the open space. It shows that such an oscillator would, generally, oscillate at multiple frequencies. The spectrum of oscillations is not identical to the spectrum of intrinsic frequencies of the relevant passive resonator and depends crucially on the parameters of active elements (e.g., on their admittance). In a nonlinear mode the spectrum is formed self-consistently and is not governed by zero impedance condition, though the latter seems to be a reasonably good approximation at small amplitudes.

# 10. Acknowledgement

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