Approximate Wave Modeling of a Two-Dimensional Convex Dielectric Lens

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Abstract: *A full-wave solution to the problem of electromagnetic wave focusing with two-dimensional convex dielectric lens of canonical cross-section formed by two circular arcs. Focusing effect is simulated for two polarizations of incident plane waves, TE and TM. Based on these simulations, an approximate method is proposed to reproduce the full-wave solution with reasonable accuracy for lenses of relatively large size. Along with the plane wave focusing, we consider the effects of point-to-point focusing (quality of image formation) and far-field beam shaping when the point source is located at the lens focus. Numerical results are presented to show the energy density profiles for both full-wave and approximate analysis.*

1. Introduction

There is a need for the accurate wave-like asymptotic modeling of large-scale quasi-optical systems with dielectric lenses. A typical example of system of this kind is QUaD, a sub-millimeter-wave telescope for Cosmic Microwave Polarization experiment on DESI site at South Pole [1,2]. The very low level of expected signals requires a detailed study of spurious effects introduced by optical elements of the system. The problem is particularly acute when the fine structure of radiation field in the focal domain is of major importance or polarization effects have to be computed with relatively high precision. In this case, the problem requires advanced methods for lens simulations analogous to physical optics modeling of reflector antennas [3]. When developing methods suitable for accurate asymptotic simulations, comparisons with exact solutions are of great importance. In the meantime, exact simulation of electromagnetic wave scattering by a dielectric body of arbitrary shape is a complicated problem, especially, for systems of large electrical size as compared to the radiation wavelength.

There are various approaches to numerical solution of dielectric body scattering problems kind ranging from relatively straightforward finite difference, finite elements, and similar kinds of methods [4] to special sophisticated formulations based on the integral equations [5,6] combined with regularization methods of their solution [7-9]. Exact methods are usually limited to systems of small size, often less than ten wavelengths in diameter even in those cases when advanced solvers are used (e.g., Ansoft HFSS software). Besides, there is limited number of exact solutions for dielectric lenses described in the literature.

One example is a recent simulation of a convex micro-cylindrical axial-lens [10] by using the integral equation method [6] with no regularization. A rigorous solution based on regularization technique is presented in [8,9] for a special class of small-size two-dimensional problems of relatively complicated geometry. A promising hybrid method is proposed for diffractive micro-lenses in [11] where the finite difference method solves the problem in the near field of the lens and the partial plane wave propagation is used to transform the solution to the far-field domain.

In general, however, there are no analytical solutions published for typical canonical lens geometries, e.g., for common two-sided convex lenses of conventional shape. Similarly, there are no advanced asymptotic methods developed for these problems so that the methods were both accurate and efficient, exceeding the capabilities of usual ray tracing approach. Meanwhile, it is this kind of problems that needs both the full-wave solutions and the enhanced asymptotic simulations in practical applications.

2. Analysis

A solution is obtained in terms of cylindrical wave expansions of fields with respect to the frames associated with the circular arcs forming the lens cross-section profile. The Neumann addition theorem for cylindrical functions is used for transforming the expansions from one frame to another when satisfying the boundary conditions for tangential fields at the lens surfaces. Because of rapid growth or decay of cylindrical functions with the angular harmonic index, the solution of this kind is limited to lenses of relatively small size. Nevertheless, using a quadruple precision arithmetic, we could simulate lenses with the surface curvature radii $R_1=R_2=R$ up to $R=10\lambda$ (λ is the free-space wavelength) at the lens diameter $D=R$.

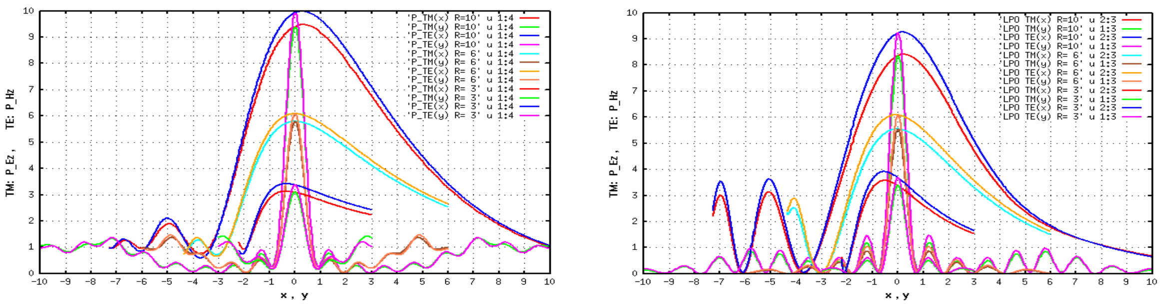
Asymptotic approximations are based on the following Kirchhoff diffraction integral:

$$U_{\mathbf{K}}(P) = \int_{S2} \left(U\frac{dG}{dn} + \frac{dU}{dn}G \right) ds$$

where U is the field on the surface $S2$ and U_K is the scattered field. G is the free space Green's function.

3. Simulations

The results showing the focusing effects in E and H polarizations are plotted in Fig. 1 (a). For convenience, we choose the lens refraction index n=1.5 so that the geometrical focal point F_1 (F_2) coincides with the frame origin O_1 (O_2). The energy density associated with the E (TM) or H (TE) field component is shown as a function of longitudinal (x) and transverse (y) coordinates as marked by legends ($x=x_1/\lambda$ at $y_1=0$, $y=y_1/\lambda$ at $x_1=0$).



(a) (b)

Figure 1. Energy density of the E (TM) or H (TE) field component in the focal domain of a convex 2D lens with plane wave illumination: (a) full-wave analysis and (b) approximate wave modelling ($R/\lambda=3, 6, 10$).

A common feature observed in all simulations is that the TE polarized wave produces greater energy density at the focal point as compared to the TM wave of the same incident power. This is the result of difference in TE and TM wave transmission at oblique incidence at the lens surfaces as expressed by the Fresnel coefficients. The second feature is that the width of the focal spot defined as the distance between the first minima of the filed is twice the wavelength while the width of fringes is one wavelength for lenses of different size, with similar patterns in both polarizations. In addition, there are some intrinsic resonant effects associated with lenses, which are especially pronounced in the narrow-band applications. With increasing the size of lens and the bandwidth of radiation, the resonant effects smear out and become insignificant.

Approximate wave modelling of a 2D dielectric lens is developed following the ideas of the physical optics approach. There could be a variety of different implementations of this approach. The results obtained in one of the models are shown in Fig. 1 (b) that should be compared with the accurate modeling in Fig. 1 (a). In general, there is a good agreement between the results. Specifically, it concerns the difference of the wave energy density in different polarizations and the size and location of the focal spot. In the same time, the energy density at the focal point shows 10% inaccuracy in these examples and the error remains unknown for lenses of greater size. Truncation of the integration domain also appears to be an issue that requires special examination.

4. Conclusions

Both full wave and approximate modelling of a 2D canonical shape dielectric lens are developed for both polarizations. The full wave model consists of the cylindrical expansion of the fields. The approximate model follows the ideas of the physical optics. Numerical results are produced and they show a good agreement between the exact and asymptotic approaches.

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